Vortex beams with strong longitudinally polarized magnetic field and their generation by using metasurfaces

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A novel method of generation and synthesis of azimuthally E-polarized vortex beams is presented. Along the beam axis such beams have a strong longitudinally polarized magnetic field where ideally there is no electric field. We show how such beams can be constructed through the interference of Laguerre–Gaussian beams carrying orbital angular momentum (OAM), and then quantify the longitudinal magnetic field of such beams. As an example, we present a metasurface made of double-split ring slot pairs and report a good agreement between simulated and analytical results. Both a high magnetic-to-electric-field contrast ratio and a magnetic field enhancement are achieved. We also investigate the metasurface physical constraints to convert a linearly polarized beam into an azimuthally E-polarized beam and characterize the performance of magnetic field enhancement and electric field suppression of a realistic metasurface. These findings are potentially useful for novel optical spectroscopy related to magnetic dipolar transitions and for optical manipulation of particles with spin and OAM. © 2015 Optical Society of America

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1. INTRODUCTION

Spectroscopy systems usually work based on electric dipole transitions, which are dominant effects in the interaction of molecules and atoms with electromagnetic fields. However, it would be desirable to boost the magnetic dipole transitions, which are weaker than the electric ones, to a level that can be directly detected. It is demonstrated in [1] that the ratio of magnetic dipole to electric dipole absorption rate is proportional to the square ratio of the magnetic and the electric field, $|H|^2/|E|^2$. Thus, detection of magnetic dipole transitions can be boosted selectively to rates comparable to electric dipole transitions by driving the particles with beams whose magnetic-to-electric-field ratio is purposely engineered. The magnetic-to-electric-field ratio is significant in the near field region of a very small circular aperture (like a fiber tip), and greatly enhances as the aperture radius decreases [2]. However, for practical aperture radii, the enhancement in magnetic field intensity is negligible [2]. For an azimuthally E-polarized beam, the magnetic-to-electric-field ratio is significantly larger than that of a plane wave $|H|/|E| = 1/\eta_0$ on the beam axis, where $\eta_0$ is the free-space wave impedance [1]. Azimuthally E-polarized beams are therefore promising for microscopy and spectroscopy methods based on detection of both magnetic and electric dipole transitions. Optical circular dichroism to study a vast amount of organic chiral molecules [3] would also benefit from enhancing magnetic fields.

Azimuthally E-polarized beams can be directly generated by coherent interference of two orthogonally polarized TEM$_{01}$ laser modes [4]. In the past decades, there also has been a growing interest in novel azimuthal and radial polarizers comprising anisotropic metallic and dielectric structures with the ability to mimic polarization manipulation capabilities of the natural birefringent media. These include interferometric techniques [5], holograms [6], liquid crystal devices [7], spatial light modulators [8], and multi-elliptical core fibers [9]. Various flat optics devices also can be realized by planar fabrication technologies that received considerable attention recently [10–12]. Space-variant gratings also have been used to convert the circularly polarized incident beams into radially or azimuthally E-polarized beams at far-infrared [13] and visible [14] ranges. Optical metasurfaces comprising nanoantennas offer vast flexibility in the design of space variant polarizers by spatially tailoring the polarization state of an incident beam. A superposition of the radial polarizer and the fork diffraction hologram are proposed in [15] to generate a radially polarized beam from a circularly polarized beam. A metasurface comprising spatially rotated linear polarizers (rectangular apertures) is also proposed in [16] to convert a circularly polarized beam into a vector beam. Recently, a setup consisting of an inhomogeneous half-wave plate meta-surface to generate vector beams is demonstrated in [17,18]. Here, we adopt an approach similar to those in [17,18], but with a different metasurface element to increase efficiency. Furthermore, the main goal is different from [17,18], because here we focus only on the properties and generation of azimuthally polarized beams for controlling and quantifying the longitudinal magnetic field.

In this paper, we examine, both analytically and numerically, the generation of azimuthally E-polarized vortex beams through interference of Laguerre Gaussian (LG) beams, and...
study the evolution of their electric and magnetic field distributions as they propagate in a host medium. Ideally, such beams possess no electric field along the beam axis where only a longitudinally polarized magnetic field is present. This characteristic is the main interest of this investigation. After showing the basic principles for generating such beams, we show how these specific beams can be generated by using metasurfaces and investigate the physical parameters they should possess. The azimuthally E-polarized vortex beam is generated from a linearly polarized incident Gaussian beam passing through a flat inhomogeneous half-wave plate metasurface made of anisotropic nanoantennas, as shown in Fig. 1. Although here we are only interested in the generation of the azimuthally polarized vector beam, the proposed metasurface can be used to generate a vector beam with any desired spatial polarization distribution on the higher-order Poincaré sphere [19]. We also show that a large magnetic-to-electric-field contrast ratio is obtained with the goal to describe only where an intense magnetic field is present. Finally, we show how focusing the generated azimuthally E-polarized vortex beam through a high numerical aperture (NA) lens provides a strong longitudinally polarized magnetic field in a narrow spot on the beam axis where the total electric field is vanishing.

2. ANALYTICAL MODEL

Consider an azimuthally E-polarized vortex beam, whose total electric field is expressed as

\[
E = E_0(\rho, z)e^{ikz}(\rho \sin \phi \hat{x} + \cos \phi \hat{y}) = E_0(\rho, z)e^{ikz}\hat{\phi}. \tag{1}
\]

Here, bold letters denote vectors, and caret (\(^\wedge\)) denote unit vectors. Furthermore, in the following we consider time harmonic fields with \(\exp(-i\omega t)\) time dependence, which is suppressed for convenience. The corresponding magnetic field is then found by \(V \times E = -i \omega \mu H\) in cylindrical coordinates, yielding

\[
\mathbf{H} = \frac{i}{\omega \mu} \left[ \frac{\partial E_0(\rho, z)e^{ikz}}{\partial z} \hat{\rho} - e^{ikz}\left( \frac{E_0(\rho, z)}{\rho} + \frac{\partial E_0(\rho, z)}{\partial \rho} \right) \hat{\phi} \right]. \tag{2}
\]

If \(E_0(\rho, z)\) has a zero of order 2 or more on the beam axis at \(\rho = 0\), both the electric and magnetic fields are zero along the beam axis. However, in a special case that \(E_0(\rho, z)\) has a simple zero at \(\rho = 0\), the longitudinal component of the magnetic field is nonzero on the beam axis where the magnitude of the total electric field is zero. Note that the longitudinal component of the electric field is zero everywhere in the paraxial regime and will be explained later on in the paper (see Appendix A). This means that there is an infinite magnetic-to-electric-field contrast ratio on the beam axis. Let us rewrite the azimuthally E-polarized field in Eq. (1) as follows:

\[
E = E_0(\rho, z)e^{ikz}\hat{\phi} = E_0(\rho, z)e^{ikz}\left[ -\frac{e^{i\phi} - e^{-i\phi}}{2i} \hat{x} + \frac{e^{i\phi} + e^{-i\phi}}{2} \hat{y} \right]. \tag{3}
\]

In Eq. (3), we can easily recognize \(e^{+i\phi}\) phase dependences which represent angular orbital momentum (OAM) carrying beams with OAM numbers \(\pm 1\) [20], as explained in the following. Thus, initial characterization of the azimuthally E-polarized beams with longitudinal polarized magnetic field along the beam axis shows that they need to possess a simple zero of radial \(E\)-field profile function \(E_0(\rho, z)\) and they are a superposition of OAM carrying beams. In order for the ideal azimuthally E-polarized vortex beam in Eq. (1) to be physical, its electric and magnetic fields should satisfy the wave equation. Let us resort for a moment to linearly polarized LG laser modes with electric field \(u(\rho, \phi, z)e^{ikz}\hat{x}\), where the complex scalar function \(u(\rho, \phi, z)\) satisfies the wave equation in cylindrical coordinates that under paraxial approximation [20] reduces to

\[
\left[ \frac{1}{\rho} \frac{\partial}{\partial \rho} \left( \rho \frac{\partial}{\partial \rho} \right) + \frac{1}{\rho^2} \frac{\partial^2}{\partial \phi^2} + 2ik \frac{\partial}{\partial z} \right] u(\rho, \phi, z) = 0. \tag{4}
\]

The solutions \(u(\rho, \phi, z)\) in cylindrical coordinates are LG beams that exhibit an angular phase variation as \(\exp(il\phi)\) and carry \(lh\) OAM per photon [20,21]. Here, \(h\) is reduced Planck’s constant and the parameter \(l\) is called OAM number or l number. These LG mode solutions are expressed as [20,21]

\[
u_{lp} = \sqrt{\frac{2\rho}{\pi(p + |l|)!}} \left[ \frac{\rho \sqrt{2|l|}}{w} \right] e^{-\frac{\rho^2}{2w^2}} \phi e^{l\phi} \times L_p^{(0)}(\frac{2\rho^2}{w^2}) e^{-\frac{1}{2}(2p + |l| + 1)\tan^{-1}(\frac{z}{R})} \phi^{l\phi}, \tag{5}
\]

where \(u_{lp}\) has unit norm,

\[
w = w_0 \sqrt{1 + (z/z_R)^2}, \quad R = z(1 + (z_R/z)^2). \tag{6}
\]

\(L_p^{(0)}(\cdot)\) is the associated Laguerre polynomial given as

\[
L_p^{(0)}(\frac{2\rho^2}{w^2}) = \sum_{m=0}^{p} \frac{(-1)^m (p + |l|)!}{(p - m)! (|l| + m)! m!} \left( \frac{2\rho^2}{w^2} \right)^m, \tag{7}
\]

\(p\) indicates the radial mode number, \(w_0\) is the beam waist of fundamental Gaussian mode [LG mode with \((l,p) = (0,0)]\), \(z_R = \pi w_0^2/\lambda\) is the Rayleigh range, and \(k = 2\pi/\lambda\) are the wavenumber and wavelength in the host medium, respectively. To ensure that the electric field distribution in Eq. (1) constitutes a self-standing beam with an infinite magnetic-to-electric-field-contrast ratio along the beam axis, (i) it must...
satisfy the paraxial wave equation (thus, it must be a linear combination of LG modes with \( l \) numbers \( \pm 1 \)); and (ii) the radial field profile \( E_\rho (\rho , z) \) must have a simple zero at \( \rho = 0 \). The transverse E-field profiles of LG modes with \( l \) numbers \( \pm 1 \) have the proportionality \( u_{\pm 1,p}/\rho^{l+|l|} \) as \( \rho \to 0 \). Note that for any \( p \), the first term of associated Laguerre polynomial \( |m| = 0 \) term of the summation in Eq. (7) is a constant equal to the combination \( \langle l^{|l|} \rangle \), which equals \( p + 1 \) when \( l = \pm 1 \). As \( m \) increases in Eq. (7), higher-order polynomial terms proportional to \( (2\rho^2/w^2)^m \) are added. Owing to the \( m = 0 \) term in Eq. (7), the radial field profiles \( u_{\pm 1,p}/\rho^{l+|l|} \) of all the LG modes with \( l = 1 \) and \( p = 0, 1, 2, \ldots \) have a simple zero as \( u_{\pm 1,p} \propto \rho \) when \( \rho \to 0 \). Therefore, one can use pairs of LG modes \( u_{1,p} \) and \( u_{-1,p} \) to construct an azimuthally E-polarized beam. By substituting the radial field distribution of LG modes with \( l = 1 \) and any arbitrary \( p \) in Eq. (3) as \( u_{\pm 1,p} = E_0(\rho, z)e^{\pm ip} \) with

\[
E_0(\rho, z) = \frac{2\rho}{w^2}\sqrt{\pi (p + 1)}e^{-\rho^2/2}e^{-2ip|l|\tan^{-1}(z/p)}L_p^{(1)}(2\rho^2/w^2),
\]

the electric field takes the form

\[
E = \frac{u_{1,p} - u_{-1,p}}{2i} \hat{x} + \frac{u_{1,p} + u_{-1,p}}{2} \hat{y} = -i\sqrt{2}(u_{-1,p}\hat{e}_h - u_{1,p}\hat{e}_h) = -i\sqrt{2}E_0(\rho, z) (e^{-i\omega t}\hat{e}_h - e^{i\omega t}\hat{e}_h).
\]

Note that, in Eq. (9) and in the following, the \( e^{ikz} \) term is suppressed for convenience. The right-hand and left-hand circularly polarized unit vectors are defined as \( \hat{e}_h = (\hat{x} + i\hat{y})/\sqrt{2} \) and \( \hat{e}_h = (\hat{x} - i\hat{y})/\sqrt{2} \). Note that this beam has the higher-order Stokes’ parameters \( S_{0}^{1} = S_{0}^{3} = -1 \) and \( S_{2}^{1} = S_{2}^{3} = 0 \), following the discussion in [17, 19]. Therefore one needs two LG beams with \( l = \pm 1 \), with opposite sense of circular polarization, to generate the required beam. Since the linearly polarized LG beams \( u_{\pm 1,p}\hat{x} \) and \( u_{\pm 1,p}\hat{y} \) are solutions to the paraxial wave equation, any linear combination of those, including the azimuthally E-polarized vortex beam expressed in Eq. (9), is a solution as well. The longitudinal electric and magnetic field components of azimuthally E-polarized vortex beam can be obtained from its transverse electric field components given in Eq. (9) by using Maxwell equations under paraxial approximation (See Eqs. (19)–(23) in [22]). It is demonstrated in Appendix A that, for an azimuthally E-polarized vortex beam as in Eq. (9), the longitudinal electric field component is zero everywhere in the paraxial regime. The longitudinal magnetic field is also obtained from Eq. (2) as

\[
H_z = \frac{-2i}{w^2\omega \mu \sqrt{\pi (p + 1)}}e^{\rho^2/2}e^{-2ip|l|\tan^{-1}(z/p)} \times \left( L_p^{(1)}(2\rho^2/w^2) \left[ 2 + \rho^2(ikR - 2w^2) \right] - 4\rho^2/w^2 S \right),
\]

where

\[
S = \begin{cases} 
0 & p = 0 \\
L_p^{(2)}(2\rho^2/w^2) & p \geq 1
\end{cases}
\]

Note that the strength of the longitudinal magnetic field \( H_z \) on the beam axis is inversely proportional to the square of beam waist as in Eq. (10), implying that a tightly focused beam can boost the longitudinally polarized magnetic field level. Figure 2 shows the magnitude of total electric field and longitudinal magnetic field at the transverse plane (\( z = 0 \)) for an azimuthally E-polarized vortex beam generated using two circularly polarized LG beams with \( l = \pm 1 \) and \( p = 0 \). The electric vector field distribution is symmetric about the beam axis as shown in Fig. 2(a). It is observed that the total electric field of azimuthally E-polarized beam vanishes on the beam axis, whereas the longitudinal component of magnetic field takes its maximum value. Note that the amplitude of the total electric field is maximum at the radius \( \rho = w_0/\sqrt{2} \). The square of the normalized magnetic-to-electric-field contrast ratio \( |\eta_0 H_z^2/|E|^2| \) versus radial distance from the beam axis is shown in Fig. 3. Note that the normalized magnetic-to-electric-field-contrast ratio tends to infinity on the beam axis because the electric field vanishes there.

3. METASURFACE-BASED AZIMUTHAL POLARIZER

In this section, a thin plasmonic metasurface capable of converting a linearly polarized incident beam into an azimuthally E-polarized vortex beam is conceived. In the previous section, we demonstrated that an azimuthally E-polarized vortex beam can be realized by interference of two circularly polarized LG modes with \( l \) numbers \( \pm 1 \). Next, we introduce in
Figure 4. Top view of an arbitrary-shaped anisotropic slot nanoantenna unit cell. (a) Reference unit cell with zero rotation angle (local and global coordinate systems coincide with each other). (b) The same unit cell rotated by \( \psi \) degrees, indicating both local (primed) and global (nonprimed) coordinate systems.
and the total transmitted wave is the sum of four terms with regard to Eqs. (14) and (15), yielding

\[ E_i' = E_{iFG}^t + E_{iLG}^t, \]  
\[ \text{(17)} \]

where the subscripts “FG” and “LG” stand for fundamental Gaussian and Laguerre Gaussian, respectively. The sum of fundamental Gaussian beams, whose phases are not controlled by the rotation angle as seen in Eq. (15), yields a linearly polarized beam:

\[ E_{iFG}^t = u_{0,0}(\rho, z) \frac{A}{2}(a_{ih}^i \hat{e}_r + a_{ih}^i \hat{e}_h) \]
\[ = u_{0,0}(\rho, z) \frac{A}{2}(a_\xi \hat{\mathbf{x}} + a_\eta \hat{\mathbf{y}}) = \frac{A}{2} E_{0FG}^i. \]
\[ \text{(18)} \]

and the sum of LG modes with \( l \) numbers +1 and −1 generated by the rotational phase control method is represented as

\[ E_{iLG}^t = E_{1}(\rho, z) \frac{B}{2} (a_{ih}^i e^{i\varphi} \hat{e}_r + a_{ih}^i e^{i\varphi} \hat{e}_h). \]
\[ \text{(19)} \]

Here, \( E_{1}(\rho, z) = E_{-1}(\rho, z) \) represents the radial field profile of the \( E_{iLG}^t \) transmitted beam, and it is composed of LG modes with \( l \) numbers ±1, and \( p = 0, 1, 2, \ldots \)

\[ E_{\pm 1}(\rho, z)e^{i\varphi} = \sum_{p=0}^{\infty} a_{\pm 1,p} u_{\pm 1,p}, \]
\[ \text{(20)} \]

as shown in Appendix B. In the case with fundamental Gaussian mode incidence as investigated here, the modes with \( p = 0 \) (with the coefficients \( a_{\pm 1,0} \)) will be the dominant modes in Eq. (20). Note that \( u_{1,p}/e^{i\varphi} = u_{-1,p}/e^{-i\varphi} \); hence from Appendix B it can be shown that \( a_{1,p} = a_{-1,p} \).

Equation (17) represents the two contributions to the total transmitted wave. The first one is a linearly polarized beam, whereas the second one is what we want to generate for obtaining beams with space-variant polarization. It is convenient to represent the radial and azimuthal components of the total transmitted wave \( E' = E'_r \hat{\mathbf{r}} + E'_\varphi \hat{\varphi} \) as

\[ E'_r = \frac{1}{2}[A u_{0,0}(\rho, z)(a_{rh}^i e^{i\varphi} + a_{rh}^i e^{-i\varphi}) + B E_{1}(\rho, z)(a_{rh}^i + a_{rh}^i)] \]
\[ E'_\varphi = \frac{i}{2}[A u_{0,0}(\rho, z)(a_{rh}^i e^{i\varphi} - a_{rh}^i e^{-i\varphi}) + B E_{1}(\rho, z)(a_{rh}^i - a_{rh}^i)]. \]
\[ \text{(21)} \]

For a purely \( y \)-polarized incident wave (i.e., with \( a_{rh}^i = -a_{rh}^i \)), when \( T_y = -T_x \) (thus \( A = 0, B = 2T_x \)), it is clear from Eq. (21) that \( E'_r = 0 \) and an azimuthally E-polarized vortex beam is obtained. This is also confirmed from Eq. (18) where the fundamental Gaussian beam contribution vanishes, and from Eq. (19) where the LG contribution takes the form of Eq. (9) describing an azimuthally E-polarized beam. On the other hand, for a \( x \)-polarized incident wave with \( a_{rh}^i = a_{rh}^i \), when \( T_y = -T_x \), a pure radially polarized beam is obtained.

Our goal is to show that one can achieve high magnetic-to-electric-field contrast by creating an azimuthally E-polarized beam under a \( y \)-polarized incident wave. In practice, guaranteeing \( A = 0 \) is not realistic, whereas one can implement \( |A| \ll |B| \) and create a mainly azimuthally polarized beam. For this case, because of the interference of the \( y \)-polarized \( E_{0FG}^i \) and the \( y \)-polarized \( E_{0LG}^i \) contribution, the transverse electric field null does not appear at \( \rho = 0 \) and slightly shifts on the \( x \) axis (where \( \varphi = \pm \hat{\varphi} \) and \( E'_\varphi = 0 \)). By setting \( E'_\varphi = 0 \) on the \( \pm x \) axis, the null transverse E-field location is found by solving

\[ \pm A u_{0,0}(\rho, z) + B E_{1}(\rho, z) = 0. \]
\[ \text{(22)} \]

When \( |A| \ll |B| \), one can still realize an extremely small E-field close to the beam axis. On the other hand, there is a strong longitudinal magnetic field close to the beam axis because of the generation of azimuthally E-polarized beam \( E_{LG}^i \) in Eq. (19). Note that all the LG contributions in Eq. (20) with \( l \) numbers ±1, and \( p = 0, 1, 2, \ldots \), contribute to the strong \( H_z \) on the beam axis as explained in Section 2. Thus, the strength of total \( H_z \) on the beam axis \( (\rho = 0) \) is found with a summation as

\[ H_z = \frac{2B}{u_p^2 a_{\rho,\varphi} \sqrt{\pi}} (a_{rh}^i - a_{rh}^i) \sum_{p=0}^{\infty} a_{1,p} \sqrt{(p + 1) e^{-[(p + 1)\tan^{-1}(\rho/z)]}}. \]
\[ \text{(23)} \]

4. DESIGN OF A METASURFACE

The proof of the proposed concept is shown in this section for a flat azimuthal polarizer metasurface of radius 20\( \lambda \) designed to operate at \( \lambda = 6 \) \( \mu \)m to convert the linearly polarized incident Gaussian beam to an azimuthally E-polarized vortex beam. A double-layer double-split ring slot element illustrated in Fig. 5 is adopted to satisfy the \( T_{xy} = -T_y \) condition required for generation of an azimuthally polarized beam. Two ring slots, one rotated by 90° relative to the other, are independently excited by orthogonal linear polarizations. Depending on the chosen dimensions, the two polarizations experience distinct resonance frequencies. By tuning the physical parameters, the condition \( T_{xy} \approx -T_y \) can be realized at the operating frequency, which lies between the two properly designed resonance frequencies. The double-layer cell is chosen because of its higher transmission amplitude than that of the single-layer cell [27]. The array element with rotation angle \( \varphi = 0 \)° is characterized in an infinite array setup under normal incidence using the finite element method implemented in CST Microwave Studio frequency domain solver, and its amplitude and phase of transmission coefficients are plotted in

Fig. 5. (a) Top view, and (b) 3D view of a double-layer double-ring slot resonator: \( r_1 = 0.63 \text{ \( \mu \)m} \), \( r_2 = 0.79 \text{ \( \mu \)m} \), \( r_3 = 0.9 \text{ \( \mu \)m} \), \( r_4 = 1.06 \text{ \( \mu \)m} \), \( \alpha_1 = 66° \), \( \alpha_2 = 20° \), and \( h = 1 \text{ \( \mu \)m} \), \( p = 2.4 \text{ \( \mu \)m} \).
Fig. 6 versus frequency. It is observed that, at the operating frequency (50 THz), the transmission coefficients for $x'$- and $y'$-polarized waves are of equal amplitude (with the insertion loss of 4.6 dB) and have a $164^\circ$ phase difference. This characteristic means that $T_{x'} \approx -T_{y'}$, i.e., $|A| = 0.1697$ and $|B| = 1.2072$ with $|A| \ll |B|$. Hence, the resultant transmitted beam will be mainly azimuthally E-polarized with weight $B$, and there will be a remnant of Gaussian beam with linear polarization in the transverse plane, with weight $A$. In general, other unit cell elements can be used, considering also more metasurface layers, to further decrease the insertion losses at mid-infrared and to further minimize the ratio $|A|/|B|$.

Based on the phase control method described in Section 3, the transmission characteristics of rotated elements are derived from the characteristics of reference one with $\psi = 0^\circ$. One can estimate the transmitted field at every cell location by resorting to the concept of rotation and local periodicity discussed in Section 3, without the need of characterizing all possible unit cell configurations and the whole metasurface in full-wave simulation environment. The transmitted field through the metasurface polarizer is approximated as piece-wise constant distribution over each cell on the metasurface.

In the following, we utilized a 2D forward and inverse Fourier transform implemented numerically to model the propagation of transmitted beam through the metasurface. The transmitted electric field over a cell at a very short distance of the metasurface is approximated with uniform distribution over each unit cell and evaluated as $E_{\text{cell}}(\rho_{\text{cell}}, \varphi_{\text{cell}}) = T(\rho_{\text{cell}}, \varphi_{\text{cell}})/2 \cdot E_0(\rho_{\text{cell}}, \varphi_{\text{cell}})$, where $T(\rho_{\text{cell}}, \varphi_{\text{cell}})$ is the transmission tensor of the nanoantenna element rotated by the angle $\psi$, and $(\rho_{\text{cell}}, \varphi_{\text{cell}})$ is the position of the center of a cell. This is the field profile over the metasurface that generates the beam as in Eq. (17) further away from the surface. Propagation of the transmitted field through the metasurface is determined by first Fourier transforming the transverse piece-wise approximated electric field just over the transmission side of the polarizing metasurface assumed to be at $z = z_0$. This spectral transverse E-field is evaluated based on the Fourier transform formula:

$$\tilde{E}(k_x, k_y, z_0) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} E(x, y, z_0) e^{-ik_x x - ik_y y} dx dy.$$  \hspace{1cm} (24)

Then the field is reconstructed by the inverse Fourier transform at any arbitrary transverse plane as

$$E(x, y, z) = \frac{1}{4\pi^2} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} [E(k_x, k_y, z_0)e^{ik_x x + ik_y y} dk_x dk_y].$$  \hspace{1cm} (25)

where

$$k_z = \left\{ \begin{array}{ll} \sqrt{k_0^2 - (k_x^2 + k_y^2)} & \text{if } k_0 \geq (k_x^2 + k_y^2) \\ i \sqrt{(k_x^2 + k_y^2) - k_0^2} & \text{if } k_0 < (k_x^2 + k_y^2) \end{array} \right.$$  \hspace{1cm} (26)

The double integrals in Eqs. (24) and (25) are efficiently calculated by using a 2D FFT algorithm, where the size of the entire spatial domain is $81.92 \lambda \times 81.92 \lambda$ with spatial resolution of $\lambda/50$. The metasurface located at $z = z_0$ covers a circular area with a diameter of $40 \lambda$ and the transverse E-field is set to zero outside the metasurface area. Note that the evanescent near-field components are significant at distances very close to the metasurface. The incident wave is a linearly polarized Gaussian beam, and its electric field and power density at the beam center are $1 \text{ V/m}$ and $1.3 \text{ mW/m}^2$, respectively. The total incident beam power is $3 \times 10^{-11} \text{ W}$. The beam waist of the incident Gaussian wave is set equal to the radius of the azimuthal polarizer metasurface ($20 \lambda$) so that 80% of the total incident beam power illuminates the metasurface. Via numerical implementations of the plane wave spectrum computations [Eqs. (24) and (25)], we first show the generated azimuthally E-polarized beam’s field intensities and its evolution, and then the same beam is focused using a lens, to further boost the magnetic-to-electric-field contrast.

In Figs. 7(a) and 7(b), obtained via numerical calculations, we report the intensities of total electric field and longitudinal magnetic field (polarized along the propagation axis) of the azimuthally polarized beam generated by the metasurface on three transverse planes $z = 0.5 \lambda$, $5 \lambda$, and $10 \lambda$, respectively. The beam clearly has an electric field null and a hot spot of the longitudinal magnetic field at the center on all transverse planes. These features are broadened in space as the azimuthally polarized beam propagates and diverges. Moreover, we report in Fig. 7(c) the polarization ellipses (i.e., the trajectories of the time-domain electric field vector tip during a cycle) at several locations (superimposed to the intensity map) on the $z = 5 \lambda$ transverse plane. We also plot the phase of $E_y$ on this plane in Fig. 7(c). The electric field has a slight polarization ellipticity (deviation from a purely linear polarization) because of the “leakage” of the incident linearly polarized Gaussian beam through the metasurface as in Eq. (18). A detailed discussion on this phenomenon is provided later on in a discussion regarding Fig. 10. Nonetheless, the azimuthally polarized electric field component is dominant as indicated by the long axes of the ellipses. As shown on the right panel, the azimuthal E-field component on a constant radius is almost in phase (a mere $20^\circ$ phase variation is observed); this constitutes clear proof of the presence of the azimuthally polarized beam. Next, we investigate the features of an azimuthally E-polarized vortex beam when a focusing lens of radius $20 \lambda$ is placed $0.5 \lambda$ away from the polarizer surface. To focus the azimuthally E-polarized beam, a hyperboloidal phase profile is added to the spatial field distribution on the lens plane [28]. The intensity of the focused beam is then found in.
any transverse plane by numerically implementing the plane wave spectrum computations as in Eqs. (24) and (25). In Fig. 8, the magnitude of the total electric field (left), the longitudinal magnetic field (middle), and the normalized ratio of the total magnetic field to total electric field (right), of the focused azimuthally polarized beam at the lens focal plane are reported using lenses with different NAs. The on-axis zero electric field and annular electric field intensity distribution characteristics are clearly observed in Fig. 8, in consistency with the electric field intensity distribution results reported in [29,30], in which only the electric field intensity distribution of a tightly focused azimuthally polarized beam is examined. In contrast, here we provide a comprehensive investigation and discuss properties of the longitudinal magnetic field component of a tightly focused azimuthally polarized beam. The resultant tightly focused azimuthally E-polarized vortex beam creates a strong longitudinally polarized magnetic field in a very narrow spot (vortex region) where the magnitude of the total electric field is negligibly small. By duality, this is analogous to the radially polarized beam for which the longitudinal electric field component is strong in a narrow spot [1,31].

Note that the electric field null slightly shifts away from the exact origin; this is because of the “leakage” of the original linearly polarized Gaussian beam in Eq. (18) through the meta-surface as we will discuss in details regarding Fig. 10. Focusing the azimuthally E-polarized vortex beam through lenses with NA of 0.45 and 0.7 enhances the magnetic field levels up to 7.2 and 18 times, respectively. Moreover, this results in boosted levels of $|H_0/E|$ up to 7.2 and 18 times, respectively. The focal length of the lens with NA = 0.45 ($f = 40\lambda$), and (b) NA = 0.7 ($f = 20\lambda$). The magnetic-to-electric-field contrast ratio is normalized to its value for plane wave.
implementation made of an array of double split ring slots. Focusing the resultant azimuthally E-polarized vortex beam significantly boosts the magnetic-to-electric-field-contrast ratio in a narrow spot on the propagation axis. The performance is limited by the fundamental Gaussian beam suppression as the beam traverses through the metasurface. Such beams may find interesting applications in the optical manipulation of particles with optical magnetic polarizability. It may also open the way to future spectroscopy systems based on magnetic dipole transitions.

APPENDIX A: VANISHING LONGITUDINAL COMPONENT OF ELECTRIC FIELD FOR AN AZIMUTHALLY E-POLARIZED VORTEX BEAM

We derive here the longitudinal electric field component of an azimuthally E-polarized vortex beam. The transverse electric field of an azimuthally E-polarized beam given in Eq. (9) is rewritten as

$$E' = E_x \hat{x} + E_y \hat{y},$$

(A1)

where $x$- and $y$-components of electric field are

$$E_x = \frac{i}{2} (u_{1,p} - u_{-1,p}), \quad E_y = \frac{1}{2} (u_{1,p} + u_{-1,p}).$$

(A2)

By defining $u' = u_{1,p}/e^{i\phi} = u_{-1,p}/e^{-i\phi}$, the azimuthally E-polarized electric field in Eq. (A1) can be rewritten as

$$E = u'(\rho, z)(-\sin \phi \hat{x} + \cos \phi \hat{y}).$$

(A3)

The longitudinal electric field component can be then found from the transverse electric field components using Maxwell equations under paraxial approximations as [22]

$$E_z = \frac{i}{k} \left( \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} \right)$$

$$= \frac{i}{k} \left( - \frac{\partial (u'(\rho, z) \sin \phi)}{\partial x} + \frac{\partial (u'(\rho, z) \cos \phi)}{\partial y} \right).$$

(A4)

After taking the derivatives and using the chain rule $(\partial u'/\partial x) \times (\partial \phi/\partial x)$, Eq. (A4) is further simplified into

$$E_z = \frac{i}{k} u'(\rho, z) \left( \frac{\partial \cos \phi}{\partial y} - \frac{\partial \sin \phi}{\partial x} \right)$$

$$+ \frac{i}{k} \frac{\partial u'(\rho, z)}{\partial \rho} \left( \cos \phi \frac{\partial \rho}{\partial y} - \sin \phi \frac{\partial \rho}{\partial x} \right) = 0,$$

(A5)

which demonstrates that the longitudinal component of the electric field is zero everywhere in the paraxial regime. Note that this salient feature of azimuthally E-polarized beams is valid in the paraxial regime regardless of the radial mode number $p$ of constituent LG modes with $l = \pm 1$.

Fig. 10. Square of the normalized magnetic-to-electric-field contrast ratio of a tightly focused azimuthally E-polarized beam as in Fig. 8 for the two lenses considered. The field is evaluated on the focal plane of each case, as a function of radial coordinate, for $\phi = 0^\circ$.
APPENDIX B: PROJECTION OF TRANSMITTED FIELD ONTO LG MODES

We show how to calculate the radial field profile of the transmitted wave composed of higher-order LG modes generated by manipulating the phase distribution of the incident fundamental Gaussian mode $u_{0,0}$ [given in Eq. (5) for $(l,p) = (0,0)$] upon transmission through a proper surface. When an azimuthal phase profile $e^{i\phi}$ is added upon phase manipulation through a surface, one has a total field equal to $u_{0,0}e^{i\phi}$ which in itself does not constitute an individual LG mode solution of the paraxial wave equation. For example, LG modes of order $l_1$ are characterized by a phase distribution of $e^{i\phi_1}$; however, the field radial profile of LG modes of order $l_1$, $u_{l_1,p}$, differ greatly from that of the incident fundamental Gaussian mode $u_{0,0}$. The field profile $u_{0,0}e^{i\phi}$, on the other hand, generates all the LG modes $u_{l_1,p}$ with $p = 0, 1, 2, \ldots$. Therefore, the total field profile of generated beam is represented as

$$u_{0,0}e^{i\phi} = \sum_{p=0}^{\infty} a_{l_1,p} u_{l_1,p},$$

(B1)

which is a weighted summation of those LG modes $u_{l_1,p}$, with mode coefficients $a_{l_1,p}$. The mode excitation coefficients $a_{l_1,p}$ are found by taking the projection of transmitted field profile $u_{0,0}e^{i\phi}$ onto the LG modes $u_{l_1,p}$ owing to the orthonormality of LG modes [32]

$$a_{l_1,p} = \int_0^{2\pi} \int_0^\infty u_{0,0}e^{i\phi} u_{l_1,p} r dr dp.$$

(B2)

Note that because of the orthogonality of LG modes, no LG mode with $l \neq l_1$ can be generated by a phase profile $u_{0,0}$. As used in Eq. (19), the established beam’s amplitude distribution $E_l(r, z)$ can be defined using

$$E_l(r, z)e^{i\phi} = \sum_{p=0}^{\infty} a_{l_1,p} u_{l_1,p}.$$

(B3)

In general, the coefficients $a_{l_1,p}$ depend strongly on the initial field profile impinging on the azimuthal polarizer metasurface (here taken as $u_{0,0}$); moreover, they can be calculated numerically for any arbitrary incident field profile. Note that reversing the azimuthal phase profile added to the original field profile, which in turn becomes $u_{0,0}e^{-i\phi}$, would result in the same radial field profile $E_{-l_1}(r, z) = E_l(r, z)$ which can be concluded easily using the identity $u_{l_1,p}e^{i\phi} = u_{l_1,p}/e^{-i\phi}$ in Eqs. (B2) and (B3).

In a more general setting, the azimuthal polarizer metasurface also scales the field strength as it manipulates the phase profile of transmitted beam. These coefficients are provided for the specific case in Eq. (19).

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